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CONSIDERATIONS ABOUT MATHEMATICS

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ABSTRACT

This little book is based on the afternoon talk I gave in January 1955 to members of the Royal Society of Canada in Toronto. This society unites not only scientists, and my task was rather difficult, since there were probably only two or three mathematicians among those present. It is difficult to prepare a report for a mathematical audience that requires that it be well understood, but at the same time sufficiently informative. But how much more difficult it is to interest an audience of non-mathematicians!

Keywords: Mathematics, reflections, finding integers, the formation of mathematics, confessions of mathematics.

I. What is Mathematics?

First of all, what is mathematics? Many mathematicians, logicians and philosophers have given definitions that, in my opinion, cannot be satisfactory, especially for those who are non-specialist in mathematics. I will not try to make it better. It is quite difficult to define even one branch of mathematics. The term "geometry" is familiar to everyone, but not everyone knows its meaning. This word, especially in earlier times, was often used by many as a synonym for mathematics, and it was spoken of geometry in the same sense in which we speak of mathematics. An outstanding geometer at Princeton University, Professor Veblen defined geometry as exactly that part of mathematics that should be called geometry in the opinion of most experts (i.e., the name is established by a majority of votes - Ed.). Nowadays, many mathematicians view geometry as a branch of algebra. But the very concept of "algebra" has undergone a

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dramatic change over the past century. For many years there was a discipline known as "algebra", about the content of which there was no doubt. Then it absorbed a significant part of the theory of invariants and group theory and became "higher algebra" or simply "algebra". Its significant development over the past thirty-five years has led to the emergence of the term "abstract" algebra or "modern" algebra. And no one can now predict how long it will remain "modern" or "abstract."

So what is mathematics? It has so many different aspects that it is as difficult to define it as it is to formulate a criterion for classifying a living organism as animals or plants. Sometimes it is impossible to say with certainty whether the given theorems and their derivation belong to mathematics or whether they relate to logic. Perhaps it will be clearer and more useful if I say that mathematics probably begins where the concept of a number arises, usually in an explicit but sometimes implicit form, and in some cases so hidden that it can only be discovered as should have thought. The concept of number can arise in different ways, for example, when counting, as a means of quantitative assessments of material objects, as an abstraction of all possible measurements, as a concept associated with natural phenomena. So, geometry has its origins in problems that have arisen when measuring land. The natural curiosity of man awakened in him a desire to have a reliable logical foundation under the empirical results that were well known to him. Further reflections on this topic led to the birth of ancient Greek geometry.

Even before that, man had learned to think about numbers outside of connection with the objects from which they originated. I will not try to define the natural numbers 1, 2, 3, etc., but simply repeat the phrase of Kronecker, who said: “God created natural numbers. Everything else is the work of human hands. " For this, a person needed inquisitiveness, the ability to observe and experiment, the power of reason and logic. Some properties of natural numbers were studied at a very early stage. Euclid in his IX book examines the properties of even and odd numbers, establishing, for example, that the sum of two even and two odd numbers is an even number. But even purely practical problems forced the study of further properties of numbers. Take the famous Pythagorean result relating the lengths of the sides of a right-angled triangle. In the simplest case, the sides of a triangle are 3: 4: 5, and this is a very useful result for plotting a right angle. The question inevitably arose - are there other right-angled triangles with integer sides, and soon a triangle with a ratio of 5: 12: 13 was discovered. Next, we come to the problem of finding integers x, y, z

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such that the equality x2 + y2 = z2. It can be shown by elementary means that the solution to the problem is given by the relations:

$$x = a^2 - b^2, \quad y = 2ab, \quad z = a^2 + b^2,$$

where a and b are any integers; any number of solutions can be extracted from here. But now, of course, the following question arises: are all integer solutions satisfying the condition that x, y and z do not contain a common factor, and y is even, are covered by our formulas? This is a real mathematical problem, the solution of which is related to the arithmetic properties of numbers, and although the ancient Greeks knew these properties, they did not solve the problem. Priority here belongs to the apparently unknown Arab mathematician who wrote the manuscript dated 972 AD. Other questions arose concerning squares and cubes, and many of them could be answered with the help of those elementary methods that were then known. Many results were obtained in the fourth century AD by Diophantus, who wrote a whole book on this topic. By the Diophantine equation we mean now the equation f (x, y) = 0, if we are interested in its rational solutions - either integer or fractional. The problem of this equation has a long history, and only in our century have the most significant results related to it been obtained.

I want to emphasize once and for all that the term "mathematics" is used in this book in the sense that in Great Britain they usually put in the term "pure mathematics", as opposed to applied mathematics, natural philosophy and theoretical physics; the term "mathematician" is used accordingly. Our main interest will be associated with the results, theorems, methods and proofs of certain propositions, and not with their practical applications or considering the phenomena of the external world. Sometimes it is useful, convenient and more visual to express a mathematical problem in terms of material objects. But even when material processes are being investigated, many people are most interested in the mathematical side of the matter; such people should be classified as mathematicians. It is well known that it is precisely this interest that leads to the fundamental development of pure mathematics. Others are most interested in the material world; they view mathematics as an extremely valuable and powerful research tool. I hardly need to explain that one of the most important tasks of mathematics is to help other sciences.

The assertion has already become generally accepted that those sciences are developing most rapidly, the fundamental results of which can be formulated

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mathematically. Using mathematical methods, they deduce the most important consequences that could hardly have been obtained in any other way. This alone, not to mention other aspects, justifies the claim of mathematics to the title of Queen of Sciences.

II. Becoming a mathematician

How do you become a mathematician? There are probably more opportunities to become interested in mathematics than any other science, and many outstanding mathematicians showed undeniable signs of mathematical genius at a very early age. In elementary school, all children learn arithmetic. This is, of course, a simple form of mathematics, but there are great possibilities in it. For example, the addition of integers can prompt the problem of finding the sum of the series 1 + 2 + 3 + ... Gauss, one of the greatest mathematicians in the world, as a child, discovered a method for finding this sum, which, in fact, is used when summing an arithmetic progression. Further, such well-known concepts as line, triangle and square are fraught with the possibilities of geometric discoveries. They say that Pascal at the age of twelve independently discovered part of the geometry of Euclid,

Mathematics commonly taught in high schools in the United States and Canada, namely algebra, elementary geometry, and trigonometry, sometimes has a stimulating effect on capable students. But in most cases, students from these two countries do not show interest in mathematics until the second or third year of college.

Teaching analytical geometry and analysis has long been practiced in British second-grade schools for students capable of mathematics. Moreover, such students usually get good math practice while preparing for the entrance exams. By the time they go to university, their attraction to mathematics and their desire to work in this field are already quite pronounced.

It is interesting to recall that Isaac Newton, entering Trinity College in 1661 at the age of nineteen, knew almost no mathematics, except perhaps arithmetic.

Sometimes mathematics textbooks, even very old ones, if they accidentally catch the eye of a student, can infect him with a desire to study the subject in more detail. The Indian mathematician Ramanujan's imagination was awakened by reading the most difficult chapters of Loni's Trigonometry. My interest in algebra was awakened at thirteen or fourteen years old when I found an old American algebra textbook in a famous second-hand bookstore in Philadelphia — probably Fiklin's Complete Algebra Course, published in 1875. It is inexplicable why a book can so attract a child, make such a strong impression on him and have an impact on his

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development. His passion grows when he manages to understand the content of the book, despite all the difficulties, and his success inspires him to further attempts. Often he is quick to solve a problem that requires a lot of time and effort from his school friends or does not give in to them at all. He begins to understand the beauty of mathematics more and more, his interest grows, and soon a huge variety of mathematics opens up to him. Then he can no longer resist his attraction and makes mathematics his main occupation.

His decision is strengthened after successfully passing the exams. However, some great mathematicians, such as Galois and Hermite, did not shine in exams. And more recently, the proof of the conjecture about the approximation of algebraic numbers, which many of the strongest mathematicians in the world were looking for, was given by a man whose Cambridge record book was by no means as excellent as one might suggest from its outstanding result. Apparently, in order to pass exams successfully, some qualities are needed, which may not be possessed by an innate mathematical talent.

On the other hand, excellent exam grades cannot always predict a brilliant mathematical future. It happens that someone who showed great promise in school and college is helpless when faced with serious and creative problems. At the same time, some realize that mathematics is not their element. Thus, in the past, a number of Cambridge University students, whose work was awarded a special distinction, later, judging by the importance of their work, did not do anything worthwhile in mathematics.

But no one will go far in mathematics and become a real mathematician if they do not have some of the necessary qualities. Faith, Hope and Curiosity should live in him, and the most important of these qualities is Curiosity. He must constantly ask himself - why, how and when, and this should be the mainspring that moves him. He must believe in his abilities, in his strength and hope for success. He should never despair, but should always go forward and not allow himself to indulge in despondency for a long time. Nobody needs him anymore for the proverb: "if you didn't succeed right away, try, try, try again."

We have already said that a child, starting to study mathematics at school, can seriously get carried away with it. Therefore, it is not surprising that mathematicians often mature as researchers much earlier than representatives of other sciences, and make great discoveries before the age of twenty or before reaching thirty. They say that at this age they do their best work, and therefore math is called "the fun of the young." Indeed, many mathematicians in their younger years were able to make a

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great contribution to mathematics and solve problems that for a long time did not give in to the older mathematicians. There are many striking examples of this kind. Consequently, mathematicians can be expected to receive earlier recognition than other scientists. This is true, and is attested to by the middle age at which a mathematician receives a professorship or is elected to the Royal Society.

Hardy in his "Confessions of a Mathematician", speaking of the age at which a mathematician does his best work, writes that he does not know of a single example of major mathematical work belonging to a person over fifty years old. Much depends on the definition of the term “big work”. But I can say that already after fifty years, in 1940, I made a discovery that many young mathematicians would gladly chalk up to their own account. About two hundred years ago, Lagrange found the best possible estimate for the minimum value of a certain binary quadratic form ax2 + bxy + cy2 for integer values x and y that are not simultaneously zero. The corresponding result for an indefinite shape was obtained by A. Markov in 1879. For the binary cubic form ax3 + bx2y + cxy2 + dy3, partial and incomplete results were found around 1858 by Eisenstein, Arndt and Hermite. For eighty years, these results were the last word in this area, because there was no way to improve them. It seemed that mathematicians had cooled off to this problem. Nevertheless, in 1940, I became interested in her and managed to get the best possible result. The proof required the use of new methods, and they turned out to be suitable for wider applications. New possibilities were discovered and became available for investigation of problems to which, it seemed, there were no approaches. All this was a powerful stimulus and marked the beginning of a new great advance in the "geometry of numbers", carried out by Mahler, Davenport, C. A. Rogers and others. In the past, almost all work dealt with convex domains, but now results have been obtained for non-convex domains, which greatly expanded our knowledge. Probably, here we can draw an analogy with the situation that was created when mathematicians, who had previously studied converging series, found a means of analyzing divergent series.

III. Difficulty learning mathematics

Some mathematical statements, such as, for example, 2 + 2 = 4, 2 × 2 = 4, are considered very easily provable or not at all needing to be proved. Many do not realize that it would be extremely difficult for them to provide satisfactory proof of these claims. They do not realize that in fact they neglect the definitions of those terms that they constantly use, as well as those assumptions that can be taken as initial ones.

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Generally speaking, students, like any random person, find mathematics a very difficult subject, whatever the word “difficult” means to them, and many people complain that they are completely incapable of mathematics. Although there are many things in this science, often trivial and easy to understand, as a professional mathematician who has devoted most of my life to mathematics, I can testify that every aspect of mathematical creativity is rife with difficulties - be it studying the results obtained by others, be it it is the solution of problems posed by others or by ourselves, whether it is the study of something new.

Learning mathematics is complicated by many factors. Keeping track of mathematical arguments, even if they are short, usually requires tremendous concentration. But often the conclusion requires a long proof containing many intermediate results. With difficulty making his way through this thicket, and not seeing any connection with the final formulation of the theorem in these results, and often forgetting them, the reader begins to gradually overwork himself with the mass of details. It takes a lot of effort to grasp the main idea of the inference and see the whole evidence.

But even when the proof is finished, there is a feeling of dissatisfaction, although from the point of view of logic everything was flawless - a similar feeling appears when reading Euclid's proofs. The reader senses some gaps. The arguments can be presented in such a way that it remains unclear why the proof is needed, what it is based on and why it was successful.

Significant difficulties are observed in the study of the most developed areas of mathematics. Many of the most important, vital work done in the last fifty years is based on new ideas, and new ideas are always extremely difficult to perceive. When you first meet them, they seem so strange, elusive and far from our usual way of thinking that you feel completely confused. All that can be done in this case is to read and re-read, and gradually the vague outlines, which seemed completely devoid of reality and even content, begin to take a more definite form, the strangeness disappears due to repeated contact, and, finally, we are able to notice in a new idea is the main thing that it has, its meaning and its strength and even its great simplicity, embedded in many arguments.

Proofs may require knowledge of several areas of mathematics, and as we read the relevant literature, we gradually retreat further and further back until we begin to lose sight of the original purpose and fall into despair. This can even happen to a mathematician who has discovered new and fundamental ideas. Further development by others of the result obtained by him may require knowledge of such sections that

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he himself neglected or considered far from his scientific interests. Then he should be content with only a bird's eye view of the promised land, open to him for others.

I myself confess to many unsuccessful attempts to fully understand or master things of particular interest to me. My mind has sometimes been unable to perceive them. There were too many threads in them that I could not knit together in any way.

IV. Difficulties arising from unsuccessful presentation of the material

This section is worth highlighting as it is impossible to overestimate the difficulties posed to the reader by an unsuccessful presentation of the material. It is not easy to present arguments in a simple, logically clear way with emphasis on important points. Sometimes the definition is not clear enough from the very beginning, and you have to guess what it really means. Many authors do not realize how necessary and important it is not to spare any effort to help the reader understand what is at stake. In Euclid's proofs the premise and corollary are given for each assertion, and one can clearly see that if A implies B and B implies C, then A implies C. If we simply state that A implies C, then it is not always easy to recover the intermediate step B. It is clear enough for the author, but the reader runs the risk of confusion. It is unwise to act those authors who offer too much to take on faith. On the other hand, however, unnecessary redundant details should be avoided and it is very difficult to find the right ratio here. Sometimes a weak result is proved with tedious detailed calculations, while the proof should be obvious to every reader. In other cases, without any explanation, statements requiring verification are given, the reader can only make sure that they are correct with great effort.

The reader strives to think economically and to minimize the effort required to comprehend the subject. Not everyone views reading as a kind of research. The reader usually wants all difficulties to be smoothed out in advance so that he can easily follow the argumentation, spending the least effort.

In addition to comprehension difficulties, there are other factors that are easier to deal with that cause mental fatigue. They can be called by the used notation system containing strange and little-known symbols, or if the symbol begins to be used long after it was defined, that is, when its meaning has already been completely forgotten. It is not easy to keep a large number of symbols in memory at once - their abundance interferes with a clear perception of the essence of the matter. New definitions should not be cited that are unnecessary and can easily be omitted. Formulas can be printed in such a way that their meaning is difficult to understand. Significant difficulties arise when the reader tries to understand the general meaning of a long equation or

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identity containing many terms with superscripts and subscripts, each of which requires careful consideration to determine its meaning. Sometimes a long equality can be converted in various ways. Each subsequent equality, if necessary, should be carefully checked by the reader to find out how the subsequent equality differs from the previous one, since the author kept silent about what he was doing. In general, it is always useful if the author treats the reader in confidence. Suppose he wants to eliminate some unknowns from several equations. If he says so, the reader will easily understand the meaning of the transformations or substitutions used. If this is done without comment, the reader will hesitate for a long time until the meaning of the calculations becomes clear from what follows.

V. How does a mathematician work?

How do mathematicians work? There is a wide variety of conditions under which they can work and the techniques they use. Some mathematicians demand complete silence, while others are calm about noise and factors that interrupt work. Some are able to work for many hours in a row, while others get tired after an hour or two sessions. Some prefer to work at certain hours, while others find it impossible for themselves. It is not easy for everyone to concentrate at certain times of the day, especially if they are not in the mood for this. The brain refuses to function, and then it is impossible to neither think nor do anything.

Mathematicians think very differently. Some people cannot do this without paper and pencil. Their thinking is not only significantly stimulated by the presence of these objects, but it is often impossible without them at all. Free reflections, of course, are not excluded, but after them pencil and paper are necessary. Other mathematicians know how with their inner gaze to cover long chains of logical inference and cumbersome calculations and see them with full clarity. Talking with another mathematician is often a stimulating factor, and this is the method that has been used to do a lot of collaborative work. Some people find it very helpful to give lectures or conduct seminars. Communication with colleagues and with students often gave me fruitful ideas, but I have not written a single joint work; I can only work alone in the quiet of my office, and this excludes co-authorship.

Intuition, insight, or whatever you call it, is an important element of research. When you are faced with a fascinating problem and delve into it, it happens that, having encountered difficulties, you suddenly become imbued with the belief that there is a path leading to their overcoming, and this path is actually being sought. This is probably what happened when the best mathematical discoveries were made.

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A new idea, however, may only be the beginning, and further effort and experimentation will be required. Mathematical experimentation is similar to experimentation in the natural sciences. The research can be tedious and entailing many unsuccessful attempts; in this case, desperate floundering and wandering in different directions begins, However, this cannot be regarded as a waste of energy and time, since many different paths are tested here and the most correct one can be found. Thus, new results gradually appear and a new theorem is proved. It is worth recalling that in order to prove that A = B, it is useful to prove first that X = Y.

Sometimes you get the feeling that you are digging a valuable ore, that it contains something, although you do not know exactly what it is and where it is. Then you begin to rush here and there without any visible benefit, but each such search can give an unexpected effect and help you find a nugget of such a size that you could not even dream of.

Many mathematicians and psychologists give a big role to the subconscious. In mathematical creativity, you often face obstacles that break all your efforts. In these cases, it is recommended to put the problem aside for a while. When you return to it after a break, it can be solved with almost no effort. It falls at your feet like a ripe apple.

I remember that when I was dealing with the very difficult problem of finding the minimum value of a cubic form in two variables, I ran into obstacles that seemed insurmountable to me at first. But, reflecting on the subject a little later, during my Sunday rest, I overpowered them so easily that I could not understand why they caused me so much grief.

VI. Using computers to solve problems

So far, we've talked about the intrinsic difficulties of solving mathematical problems. The question arises: will not the external means that our century - the age of mechanisms and electronic computers - be useful. Computers, or computers, of course, play a colossal role now, and the public is sometimes inclined to believe that they can supplant mathematicians altogether. Therefore, I would like to say a few words about the help that computers can provide in solving mathematical problems. All that a mathematician can count on when dealing with computers is the speed or extreme speed of the calculations associated with the problem he puts before the machine. He may also try new methods that, without a computer, would be overwhelming or time-consuming. No one would waste years of work testing a method that does not address superimportant problems. But it hardly needs to be reminded that the machine does not have a mind and does only what the programmer

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instructs it to do (if the assignment is such that the machine is, according to its capabilities, suitable for its execution). The computer can be compared to the genie of the Magic Lamp, performing incredibly difficult tasks in the blink of an eye, but becoming extremely dangerous if misused.

Computers can be of great use, especially in number theory. According to a program developed by a mathematician, a machine can very quickly determine whether a given number is prime. She can easily find for us whole solutions of an undefined equation and bring them down to a table. This data can lead mathematicians to a new idea, leading in turn to very useful results. For example, I once asked Dr. Miller of the Cambridge Mathematical Laboratory to use a computer to find integer solutions to the equation x3 + y3 + z3 = 3, other than the obvious solution (1, 1, 1) and solution (4, 4, –5). He and Mr. Woolet didn't have much trouble getting a decision table for even more general equations x3 + y3 + z3 = k for $-100 \le k \le 100$ and x, y, z up to 3164, and for the case k = 3 new solutions in comparison with the trivial, as well as known earlier (4, 4, –5) was not found. But they noticed that the table contains an unusually large number of solutions for k = 1, and some of them are not covered by the parametric solution

$x = 9t^4, \quad y = -9t^4 - 3t, \quad z = 9t^3 + 1,$

дающим сколько угодно численных решений при подстановке instead of t numbers 1, 2, 3, ... They were interested - why is this happening? Of course, no machine could answer such a question. Reflecting on a strange fact, I found the explanation contained in a new theorem, namely in the theorem that if a, b and c are given integers, then the equation ax3 + ay3 + bz3 = bc3 has an infinite number of integer solutions other than obtained at x + y = 0, z = c. It probably would have taken a long time to get this result, if not for the computer. This example shows in which direction the use of computers in mathematical research can go.

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