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OLIY TA'LIMDA RATSIONAL KO'RINISHDAGI ANIQMAS INTEGRALLARNI HISOBLASH USULLARI

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ANNOTATSIYA

Hozirgi kunda nafaqat umumiy o'rta ta'lim maktablarini yuqori sinflarida, balki pedagogika instituti matematika yo'nalishi talabalarini matematik tahlil fanini o'qitish dolzarb masalalardan biri hisoblanadi. Shuningdek, hosila va integlarni chuqurlashtirilgan holda o'rgatish talab qilingan muammolardan biri hisoblanadi. Jumladan, karrali ildizga ega bo'lgan yuqori darajali ratsional funksiyalarni integrallash.

Kalit so'zlar: Boshlang'ich funksiya, aniqmas integral, ostragradskiy usuli, hosila, ratsional funksiya, karrali ildiz, ko'phadning EKUBi, ayniyat, rekurent formula, noma'lum koeffitsientlar usuli.

KIRISH

Mamlakatimizda matematika 2020 yildagi ilm-fanni rivojlantirishning ustuvor yo'nalishlaridan biri sifatida belgilandi. O'tgan davr ichida matematika ilm-fanni va ta'limini yangi sifat bosqichiga olib chiqishga qaratilgan qator tizimli ishlar amalga oshirildi. O'zbekiston Respublikasi Prezidentining 2020 yil 7- maydagi «Matematika sohasidagi ta'lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to'g'risida»gi PQ-4708-sonli qarori, 2020 yil 9- iyuldagi « Matematika ta'limi va fanlarni yanada rivojlantirishni davlat tomonidan qo'llab quvvatlash, shuningdek O'zbekiston Respublikasi fanlar akademiyasining V.I. Romonovskiy nomidagi matematika Instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to'g'risidagi»gi PQ-4387-sonli qarorlari hamda Muhammad Al-Xorazmiy, Ahmad Al-Farg'oniy, Abu Rayhon Beruniy, Mirzo Ulug'bek singari ulug' ajdodlarimiz tamal toshini qo'ygan matematika fani ilm-fan va texnikaning zamonaviy tarmoqlari jadal rivojlanishi munosabati bilan hozirgi kunda oliy ta'limdagi asosiy mavzular, ya'ni integrallarni chuqurlashtirilgan holda o'qitish muhim masalalardan biri hisoblanadi. Ushbu maqola yuqorida keltirilgan qarorlarning hamda oldimizga qo'yilgan masalalarni yechishda muayyan darajada xizmat qiladi.

ADABIYOTLAR TAHLILI VA METODOLOGIYA

Yuqoridagilarni hisobga olib, biz quyidagi ko'rinishdagi ratsional integralni integrallashni bir necha hil usullarini o'rganamiz.

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$$J=\int\frac{dx}{\left(x^3+1\right)^2} \qquad [1.1]$$

1-usul: Sodda kasrlarni integrallash.

$$f\left(x\right)=\frac{P_m\left(x\right)}{Q_n\left(x\right)} \qquad [1.2]$$

Kasr ratsional funksiyaning integralini hisoblash,

$$w\left(x\right)=c_0x^k+c_1x^{k-1}+...+c_{n-1}x+c_n \qquad [1.3]$$

ko'rinishdagi ko'phadni integrallashga hamda sodda kasrlar deb ataluvchi quyidagi

$$\textbf{I.}\frac{B}{x-a},\quad \textbf{II.}\frac{B}{\left(x-a\right)^r},\quad \textbf{III.}\frac{Mx+N}{x^2+px+q},\quad \textbf{IV.}\frac{Mx+N}{\left(x^2+px+q\right)^r} \qquad [1.4]$$

$\left(r>1,B,M,N,P,q-haqiqiy\ sonlar,q-\frac{p^2}{4}>0\right)$ ko'rinishdagi sodda kasrlarni integral-lashga keltiriladi. Bu sodda kasrlarning integrallari quyidagicha hisoblanadi:I va II ko'rinishdagi kasrlar $t=x-a$ almashtirish yordamida integrallanadi:

$$\int\frac{B\,dx}{x-a}=B\int\frac{dt}{t}=B\ ln\left|t\right|+C=B\ ln\left|x-a\right|+C, \qquad [1.5]$$

$$\int\frac{B}{\left(x-a\right)^r}dx=B\int\frac{dt}{t^r}=-\frac{B}{\left(r-1\right)}\cdot\frac{1}{t^{r-1}}+C=-\frac{B}{\left(r-1\right)\left(x-a\right)^{r-1}}+C \qquad [1.6]$$

III ko'rinishdagi kasrni integrallash uchun x^2+px+q - kvadrat uch hadni ushbu

$$x^2+px+q=\left(x+\frac{p}{2}\right)^2+\left(q-\frac{p^2}{4}\right)\left(q-\frac{p^2}{4}>0\right) \qquad [1.7]$$

ko'rinishga keltirib, $t=x+\frac{p}{2}$ almshtirish va $a=\sqrt{q-\frac{p^2}{4}}$ belgilash yordamida

$$\int\frac{Mx+N}{x^2+px+q}dx=\int\frac{Mt+\left(N-\frac{MP}{2}\right)}{t^2+a^2}dt=\frac{M}{2}\int\frac{d\left(t^2+a^2\right)}{t^2+a^2}+\left(N-\frac{MP}{2}\right)\cdot\frac{1}{a}\int\frac{d\left(\frac{t}{a}\right)}{\left(\frac{t}{a}\right)^2+1}=$$

$$=\frac{M}{2}\ln\left(t^2+a^2\right)+\frac{2N-MP}{2a}arctg\frac{t}{a}+C=\frac{M}{2}\ln(x^2+px+q)+\frac{2N-MP}{2\sqrt{q-\frac{p^2}{4}}}arctg\frac{x+\frac{p}{2}}{\sqrt{q-\frac{p^2}{4}}}+C \qquad [1.8]$$

munosabatga ega bo'lamiz.

IV ko'rinishdagi kasrni integrallashda yuqoridagi $t=x+\frac{p}{2},\quad a=\sqrt{q-\frac{p^2}{4}}$

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belgilashlar yordamida

$$\int \frac{Mx+N}{\left(x^2+px+q\right)^r}\,dx=\int \frac{Mt+\left(N-\frac{MP}{2}\right)}{\left(t^2+a^2\right)^r}\,dt=\frac{M}{2}\int \frac{d\left(t^2+a^2\right)}{\left(t^2+a^2\right)^r}+\left(N-\frac{MP}{2}\right) \qquad [1.9]$$

$$\int \frac{dt}{\left(t^2+a^2\right)^r}=-\frac{1}{(r-1)}\cdot\frac{1}{\left(t^2+a^2\right)^{r-1}}+\left(N-\frac{MP}{2}\right)\int \frac{dt}{\left(t^2+a^2\right)^r}$$

ni hosil qilamiz. Keyingi integral rekurent formula orqali hisoblanadi:

$$I_{r+1}=\frac{t}{2na^2\left(t^2+a^2\right)^r}+\frac{2n-1}{2na^2}I_r \qquad [1.10]$$

Bunda $r=1$ bo'lganida

$$I_1=\int \frac{dt}{t^2+a^2}=\frac{1}{a}arctg\frac{t}{a}+C \qquad [1.11]$$

Shunday qilib, har qanday haqiqiy koeffisiyentli, haqiqiy o'zgaruvchili ratsional funksiyaning boshlang'ich funksiyasi logarifim, arctangens va ratsional funksiya orqali ifoda qilinar ekan.

MUHOKAMA VA NATIJALAR

Algebra kursidan bizga ma'lumki karrali ildizga ega bo'lgan ratsional ko'rinishdagi ifodalarni noma'lum koeffitsientlar usulidan foydalanib bir nechta to'g'ri kasrlar yig'indisi shaklida ifodalash mumkin.

[1.1] integral ostida turgan karrali ildizga ega bo'lgan ratsional ifodani ham to'g'ri kasrlar yig'indisi shaklida ifodalash mumkin.

$$\int \frac{dx}{\left(x^3+1\right)^2}=\int \frac{dx}{\left(\left(x+1\right)\left(x^2-x+1\right)\right)^2}=\int \frac{dx}{\left(x+1\right)^2\left(x^2-x+1\right)^2} \qquad [1.12]$$

$$\frac{1}{\left(x+1\right)^2\left(x^2-x+1\right)^2}=\frac{A}{x+1}+\frac{B}{\left(x+1\right)^2}+\frac{Cx+D}{x^2-x+1}+\frac{Ex+K}{\left(x^2-x+1\right)^2} \qquad [1.13]$$

Ushbu tenglikni umumiy maxrajga keltirib,keyingi

$$1=A\left(x+1\right)\left(x^2-x+1\right)^2+B\left(x^2-x+1\right)+\left(Cx+D\right)\left(x+1\right)^2\left(x^2-x+1\right)+\left(Ex+K\right)\left(x+1\right)^2$$

$$1=A\left(x^5-x^4+x^3+x^2-x+1\right)+B\left(x^4-2x^3+3x^2-2x+1\right)+C\left(x^5+x^4+x^2+x\right)+$$

$$+D\left(x^4+x^3+x+1\right)+E\left(x^3+2x^2+x\right)+K\left(x^2+2x+1\right)$$

tenglikni hosil qilamiz.Tenglikni o'ng tomonidagi qavslarni ochib,ko'pxadlarning o'zaro tengligi haqidagi xossadan foydalanib, x ning bir xil darajalari oldidagi ko-effitsientlarni teglashtiramiz.

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$$\left.\begin{array}{ll} x^5 \mid & A+C=0 \\ x^4 \mid & -A+B+C+D=0 \\ x^3 \mid & A-2B+D+E=0 \\ x^2 \mid & A+3B+C+2E+K=0 \\ x^1 \mid & -A-2B+C+D+E+2K=0 \\ x^0 \mid & A+B+D+K=1 \end{array}\right\}$$

Natijada, A,B,C,D,E,K noma'lumlarga nisbatan oltita chiziqli algebraik tenglamalar sistemasi hosil qilindi. Bu sistemani yechib, noma'lum koeffitsient- larni

$$A=\frac{2}{9} \qquad B=\frac{1}{9} \qquad C=-\frac{2}{9} \qquad D=\frac{1}{3} \qquad E=-\frac{1}{3} \qquad K=\frac{1}{3}$$

ga teng ekanligini topamiz. Demak,koeffitsientlarni [1.12] tenglika olib borib qo'yib integralni hisoblaymiz/.

$$\int\frac{dx}{\left(x^3+1\right)^2}=\int\frac{dx}{\left(x+1\right)^2\left(x^2-x+1\right)^2}=\int\frac{A}{x+1}dx+\int\frac{B}{\left(x+1\right)^2}dx+\int\frac{Cx+D}{x^2-x+1}dx+$$

$$+\int\frac{Ex+K}{\left(x^2-x+1\right)^2}dx=\int\frac{\frac{2}{9}}{x+1}dx+\int\frac{\frac{1}{9}}{\left(x+1\right)^2}dx+\int\frac{-\frac{2}{9}x+\frac{1}{3}}{x^2-x+1}dx+\int\frac{-\frac{1}{3}x+\frac{1}{3}}{\left(x^2-x+1\right)^2}dx=$$

$$=\frac{2}{9}\int\frac{dx}{x+1}+\frac{1}{9}\int\frac{d\left(x+1\right)}{\left(x+1\right)^2}-\frac{1}{9}\int\frac{2x-3}{x^2-x+1}dx-\frac{1}{3}\int\frac{x-1}{\left(x^2-x+1\right)^2}dx=\frac{2}{9}\int\frac{d\left(x+1\right)}{x+1}+$$

$$+\frac{1}{9}\int\frac{d\left(x+1\right)}{\left(x+1\right)^2}-\frac{1}{9}\int\frac{2x-1-2}{x^2-x+1}dx-\frac{1}{3\cdot2}\int\frac{2x-1-1}{\left(x^2-x+1\right)^2}dx=\frac{2}{9}\int\frac{d\left(x+1\right)}{x+1}+\frac{1}{9}\int\frac{d\left(x+1\right)}{\left(x+1\right)^2}-$$

$$-\frac{1}{9}\int\frac{d\left(x^2-x+1\right)}{x^2-x+1}+\frac{2}{9}\int\frac{d\left(x-\frac{1}{2}\right)}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}-\frac{1}{6}\int\frac{d\left(x^2-x+1\right)}{\left(x^2-x+1\right)^2}+\frac{1}{6}\int\frac{dx}{\left(x^2-x+1\right)^2}=$$

$$=\frac{2}{9}\ln\left(x+1\right)-\frac{1}{9\left(x+1\right)}-\frac{1}{9}\ln\left(x^2-x+1\right)+\frac{4}{9\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}+\frac{1}{6\left(x^2-x+1\right)^2}+$$

$$+\frac{1}{6}\int\frac{dx}{\left(x^2-x+1\right)^2}=\frac{2}{9}\ln\left(x+1\right)-\frac{1}{9\left(x+1\right)}-\frac{1}{9}\ln\left(x^2-x+1\right)+\frac{4}{9\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}+ \qquad [1.15]$$

$$+\frac{1}{6\left(x^2-x+1\right)^2}+\frac{1}{6}I_2$$

I_2 ushbu integralni yechish rekurent formuladan foydalanishga olib keladi. Belgilash kiritamiz.

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$$I_2 = \int \frac{dx}{\left(x^2 - x + 1\right)^2} \qquad [1.16]$$

Ushbu integralni integrallash uchun, avval quyidagi ko'rinishdagi integralni hisoblab olamiz.

$$I_1 = \int \frac{dx}{a^2 + x^2} = \frac{1}{a} arctg \frac{x}{a}$$

$$I_1 = \int \frac{dx}{a^2 + x^2} = \begin{bmatrix} u = \frac{1}{a^2 + x^2} & dx = dv \\ du = \frac{-2x}{\left(a^2 + x^2\right)^2} dx & x = v \end{bmatrix} = \frac{x}{a^2 + x^2} + 2\int \frac{x^2}{\left(a^2 + x^2\right)^2} dx = \frac{x}{a^2 + x^2} +$$

$$+2\int \frac{dx}{a^2 + x^2} - 2a^2 \int \frac{dx}{\left(a^2 + x^2\right)^2}$$

$$I_1 = \frac{x}{a^2 + x^2} + 2I_1 - 2a^2 I_2$$

$$I_2 = \frac{1}{2a^2}\left(\frac{x}{a^2 + x^2} + I_1\right)$$

$$I_2 = \frac{1}{2a^2}\left(\frac{x}{a^2 + x^2} + \frac{1}{a} arctg \frac{x}{a}\right)$$

Hosil qilingan natijani [1.10] integralni hisoblash uchun qo'llaymiz.

$$I_2 = \int \frac{dx}{\left(x^2 - x + 1\right)^2} = \frac{1}{2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2}\left(\frac{x - \frac{1}{2}}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(x - \frac{1}{2}\right)^2} + \frac{1}{\frac{\sqrt{3}}{2}} arctg \frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) =$$

$$= \frac{2}{3}\left(\frac{2x - 1}{2\left(x^2 - x + 1\right)} + \frac{2}{\sqrt{3}} arctg \frac{2x - 1}{\sqrt{3}}\right) = \frac{2x - 1}{3\left(x^2 - x + 1\right)} + \frac{4}{3\sqrt{3}} arctg \frac{2x - 1}{\sqrt{3}}$$

Demak, rekurent formuladan foydalanib integralini yechimini topib oldik endi [1.15] tenglikka olib borib qo'yamiz va soddalashtiramiz.

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$$\frac{2}{9}\ln(x+1)-\frac{1}{9(x+1)}-\frac{1}{9}\ln(x^2-x+1)+\frac{4}{9\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}+\frac{1}{6(x^2-x+1)^2}+$$
$$+\frac{1}{6}I_2=\frac{2}{9}\ln(x+1)-\frac{1}{9(x+1)}-\frac{1}{9}\ln(x^2-x+1)+\frac{4}{9\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}+$$
$$+\frac{1}{6(x^2-x+1)^2}+\frac{1}{6}\left(\frac{2x-1}{3(x^2-x+1)}+\frac{4}{3\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}\right)=\frac{1}{9}ln\frac{(x+1)^2}{x^2-x+1}+ \quad [1.17]$$
$$+\frac{x}{3(x^3+1)}+\frac{2}{3\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}+C$$

2-usul: Ostrogradiskiy usuli:

$$\int\frac{P(x)}{Q(x)}dx=\frac{P_1(x)}{Q_1(x)}+\int\frac{P_2(x)}{Q_2(x)}dx, \quad [2.1]$$

formula Ostrogradskiy formulasi bo'lib, bunda $Q(x)$ funksiya karrali ildizga ega bo'lib, $\frac{P(x)}{Q(x)}$ -to'g'ri kasr ratsional fuksiya, $Q_1(x)$ esa $Q(x)$ va $Q'(x)$ larning eng katta umumiy bo'luvchisi, $Q_2(x)=\frac{Q(x)}{Q_1(x)}$, $P_1(x)$ va $P_2(x)$ koeffitsientlar noma'lum darajalari esa mos holda $Q_1(x)$ va $Q_2(x)$ darajalaridan bitta kam. Noma'lum koeffitsientlar

$$\frac{P(x)}{Q(x)}=\left(\frac{P_1(x)}{Q_1(x)}\right)'+\frac{P_2(x)}{Q_2(x)} \quad [2.2]$$

ayniyatdan noma'lum koeffitsientlar usuli yordamida topiladi.

Ushbu usulda [1.1] integralni hisoblashni ko'raylik. U holda,

$$P(x)=1,\ Q(x)=(x^3+1)^2,\quad Q_1(x)=x^3+1,\quad Q_2(x)=x^3+1 \quad [2.3]$$

formulaga asosan

$$\int\frac{dx}{(x^3+1)^2}=\frac{Ax^2+Bx+C}{x^3+1}+\int\frac{Dx^2+Ex+F}{x^3+1}dx, \quad [2.4]$$

deb yozib olamiz. A,B,C,D,E,F noma'lum koeffitsientlarni topish uchun, yuqoridagi [2.4] tenglikni ikkala tomonini differensiallaymiz

$$\frac{1}{(x^3+1)^2}=\left(\frac{Ax^2+Bx+C}{x^3+1}\right)'+\frac{Dx^2+Ex+F}{x^3+1}$$
$$\frac{1}{(x^3+1)^2}=\frac{(2Ax+B)(x^3+1)-(Ax^2+Bx+C)3x^2}{(x^3+1)^2}+\frac{Dx^2+Ex+F}{x^3+1}$$

bundan

$$1=(2Ax+B)(x^3+1)-(Ax^2+Bx+C)3x^2+(Dx^2+Ex+F)(x^3+1)$$
$$1=Dx^5+(-A+E)x^4+(-2B+F)x^3+(-3C+D)x^2+(2A+E)x+B+F$$

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tenglikni o'ng tomonidagi qavslarni ochib,ko'pxadlarning o'zaro tengligi haqidagi xossadan foydalanib, x ning bir xil darajalari oldidagi koeffitsientlarni teglashtiramiz:

$$\left.\begin{array}{ll} x^4 \mid & -A+E=0 \\ x^3 \mid & -2B+F=0 \\ x^2 \mid & -3C+D=0 \\ x^1 \mid & 2A+E=0 \\ x^0 \mid & B+F=1 \end{array}\right\}$$

bo'lsa,

$$D=A=C=E=0,\ F=\frac{2}{3},\quad B=\frac{1}{3}$$

ekanligini topamiz. Noma'lum koeffitsientlarni [2.4]tenglikka olib borib qo'yamiz.

$$\int\frac{dx}{\left(x^3+1\right)^2}=\frac{x}{3(x^3+1)}+\frac{2}{3}\int\frac{dx}{x^3+1}=\frac{x}{3(x^3+1)}+\frac{2}{3}\,I \qquad [2.5]$$

$$I=\int\frac{dx}{x^3+1} \qquad [2.6]$$

Ushbu integralni hisoblab

$$I=\int\frac{dx}{x^3+1}=\int\frac{dx}{(x+1)\left(x^2-x+1\right)}$$

$$=\left[\begin{array}{l}\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2-x+1}=1\\ A=\dfrac{1}{3}\ B=-\dfrac{1}{3}\ C=\dfrac{2}{3}\end{array}\right]=\frac{1}{3}\int\frac{dx}{x+1}-\frac{1}{3}\int\frac{(x-2)dx}{x^2-x+1}=\frac{1}{3}\int\frac{dx}{x+1}-\frac{1}{3}\int\frac{\left(x-\frac{1}{2}-\frac{3}{2}\right)dx}{x^2-x+1}$$

$$=\frac{1}{3}\int\frac{dx}{x+1}-\frac{1}{6}\int\frac{d\left(x^2-x+1\right)}{x^2-x+1}+\frac{1}{2}\int\frac{dx}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\frac{1}{3}\ln(x+1)-\frac{1}{6}\ln\left(x^2-x+1\right)+$$

$$+\frac{1}{\sqrt{3}}arctg\,\frac{2x-1}{\sqrt{3}}=\frac{1}{6}\ln\frac{(x+1)^2}{\left(x^2-x+1\right)}+\frac{1}{\sqrt{3}}arctg\,\frac{(2x-1)}{\sqrt{3}}$$

topib olganimzdan so'ng [2.5] tenglikka olib borib o'rniga qo'yamiz.

$$\int\frac{dx}{\left(x^3+1\right)^2}=\frac{x}{3(x^3+1)}+\frac{2}{3}\int\frac{dx}{x^3+1}=\frac{x}{3(x^3+1)}+\frac{2}{3}\,I=\frac{x}{3(x^3+1)}+\frac{1}{9}\ln\frac{(x+1)^2}{\left(x^2-x+1\right)}+$$
$$+\frac{2}{3\sqrt{3}}arctg\,\frac{(2x-1)}{\sqrt{3}}+C \qquad [2.7]$$

Demak [1.17] va [2.7] natijalar bir xil ko'rinishga ega bo'lganini ko'ramiz. Har ikki usul ham to'g'ri natijaga olib keldi.

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XULOSA

Oliy ta'limda ratsional ko'rinishdagi integrallarni hisoblash usullari turli ko'rinishga ega bo'lib,biz ulardan ikkita usulni sodda kasrlarga keltirib integrallash va Ostragradiskiy usulini ko'rib chiqdik.Ushbu ikki usulda yuqori darajali, karrali ildizga ega bo'lgan ratsional ko'rinishdagi integralni hisoblashda qulayligi va integralning qisqa yo'l bilan hisoblash mumkinligini ko'rish mumkin.Yuqoridagi integralni Ostragradiskiy va sodda kasrlarga keltirib hisoblashda rekurrent formula va noma'lum koeffisiyentlar usulidan foydalanilgan holda integral yechimi ko'rsatilgan.Yuqori darajali karrali ildizga ega bo'lgan ratsional ko'rinishdagi integrallarni hisoblashda nafaqat sodda kasrlarga keltirib hisoblash balki Ostragradiskiy uslini ham keng qo'llaniladigan va samarali usullar qatoriga kiritish va bu usulda ko'pgina integrallarni hisoblashda foydalanish mumkin.

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