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A(*z*)-ANALITIK FUNKSIYA UCHUN LORAN QATORI VA YAKKALANGAN MAXSUS NUQTALAR. *A*(*z*)-ANALITIK FUNKSIYA UCHUN CHEGIRMA.

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ANNOTATSIYA

Ushbu maqola *A*(*z*)-analitik funksiya uchun Loran qatori va yakkalangan maxsus nuqtalarga bag'ishlangan. Yakkalangan maxsus nuqtalar, ularning turlari va maxsus nuqtalarda Loran qatoriga yoyish masalasi ko'rsatilgan.

Kalit so'zlar. Yakkalangan maxsus nuqta, Loran qatori, $A(z)$-antianalitik funksiya , A-antianalitik funksiya, maxsus nuqtalarning turlari, $A(z)$ – analitik funksiyalar uchun chegirma.

Aytaylik $D-$ soha $\mathbb{C}\cong\mathbb{R}^2$ kompleks tekislikda berilgan. Agar $z=x+iy$ bo'lsa, u holda

$$\frac{\partial}{\partial z}=\frac{1}{2}\left(\frac{\partial}{\partial x}+\frac{1}{i}\cdot\frac{\partial}{\partial y}\right) \qquad \frac{\partial}{\partial \overline{z}}=\frac{1}{2}\left(\frac{\partial}{\partial x}-\frac{1}{i}\cdot\frac{\partial}{\partial y}\right)$$

$A(z)\in C(D)$ funksiya uchun biz

$$D_A=\frac{\partial}{\partial z}-\overline{A(z)}\cdot\frac{\partial}{\partial \overline{z}}, \quad \overline{D_A}=\frac{\partial}{\partial \overline{z}}-A(z)\cdot\frac{\partial}{\partial z}$$

larni yozib olamiz.

$D\subset\mathbb{C}$ da $A(z)$-antianalitik funksiya bo'lsin va $\left(L(a,r)=\left\{|\psi(z,a)|=\left|z-a+\int\limits_{\gamma(a,z)}A(\tau)d\overline{\tau}\right|<r\right\}\right)\subset\subset D$ to'plam D sohada kompakt yotsin.

1-ta'rif. *$f(z)\in C^1(D)$ funksiya $D-$ sohada A-analitik deyiladi, agar $\forall z\in D$ nuqtalar uchun quyidagi tenglik bajarilsa*

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$$\overline{D_A} f(z) = 0.$$

2-ta'rif. *Agar* $f(z)$ *funksiya ushbu*

$$\left\{ z \in \mathbb{C} : 0 < \left| z - a + \int\limits_{\gamma(a,z)} A(\tau) d\bar{\tau} \right| < r \right\}$$

sohada (a nuqtaning o'yilgan atrofi) A*-analitik bo'lsa, u holda* a *nuqta* $f(z)$ *funksiyaning yakkalangan maxsus nuqtasi deyiladi.*

Agar $A = const$ *bo'lsa* $z = \infty$ *nuqtada yakkalangan maxsus nuqtaga ta'rif berish mumkin.*

3-ta'rif. *Agar* $f(z)$ *funksiya ushbu*

$$\left\{ z \in \mathbb{C} : \mathrm{R} < \left| z + A\bar{z} \right| < +\infty \right\}$$

sohada A*-analitik bo'lsa, u holda* $a = \infty$ *nuqta* $f(z)$ *funksiyaning yakkalangan maxsus nuqtasi deyiladi.*

Yakkalangan maxsus nuqtalarning turlari.

Aytaylik a nuqta $f(z)$ funksiyaning yakkalangan maxsus nuqtasi bo'lsin. Unda $f(z)$ funksiya

$$\left\{ z \in \mathbb{C} : 0 < \left| z - a + A(\bar{z} - \bar{a}) \right| < r \right\}$$

sohada A-analitik.

$f(z)$ funksiyaning $z \to a$ dagi limitning xarakteriga qarab yakkalangan maxsus nuqtalar turlarga ajraladi.

4-ta'rif. Agar $z \to a$ da f(z) funksiyaning limiti mavjud bo'lib,

$$\lim_{z \to a} f(z) = A \quad (A\text{-chekli})$$

bo'lsa, u holda a nuqta f(z) funksiyaning bartaraf qilinadigan maxsus nuqtasi deyiladi.

5-ta'rif. *Agar* $z \to a$ *da f(z) funksiya limiti mavjud bo'lib,*

$$\lim_{z \to a} f(\mathrm{z}) = \infty$$

bo'lsa, u holda a nuqta $f(z)$ *funksiyaning qutb maxsus nuqtasi deyiladi.*

6-ta'rif. *Agar* $z \to a$ *da f(z) funksiyaning limiti mavjud bo'lmasa, u holda a nuqta f(z) funksiyaning o'ta maxsus nuqtasi deyiladi.*

Teorema 1. Aytaylik, $f \in O_A\left(L(a,R) \setminus \bar{L}(a,r)\right)$ $R > r$ bo'lsin. U holda, f funksiyani berilgan sohada quydagi ko'rinishidagi qatorga yoyish mumkin:

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$$\sum_{n=-\infty}^{\infty} C_n \psi^n(z,a)$$

Bu yerda

$$c_k = \frac{1}{2\pi i} \int_{\partial L(a:\rho)} \frac{f(\xi)}{\psi^{k+1}(\xi,a)} \left(d\xi + A(\xi) d\bar{\xi} \right),\ r < \rho < R$$

ga teng.

$A(z)$ – analitik funksiyalar uchun chegirmalar va ularni hisoblash.

$D \subset \mathbb{C}$ sohada $A(z)$ – antianalitik funksiya, yani $\frac{\partial A}{\partial z} = 0$ biz quyida $A(z) = const,\ |A(z)| \le c < 1$ holda qaraymiz.

Tarif 7. $f(z)$ A – analitik funksiyaning $a \in D$ yakkalangan maxsus nuqtadagi chegirmasi deb, yetarlicha kichik radusli $L(a,r) := \left\{ z \in D \,|\psi(z,a)| := \left| z - a + \int_{\gamma(z,a)} A(\tau) d\bar{\tau} \right| < r \right\}$ leminskata bo‘yicha olingan integralni $2\pi i$ ga nisbatiga aytiladi.va quydagicha belgilanadi.

$$res_{\substack{A \\ z=a_k}} f(z) = \frac{1}{2\pi i} \oint_{|\psi(z,a)|=r} f(z)(dz + A(z) d\bar{z})$$

Teorema 2. $f(z)$ funksiyaning yakkalangan maxsus $a \in D$ nuqtadagi chegirmasi $L(a,r)$ leminskata bo‘yicha Loran qatoriga yoyilmasidagi minus birinchi had oldidagi koefitsentga teng.

$$res_{\substack{A \\ z=a}} f(z) = c_{-1}$$

$D \subset \mathbb{C}$ sohada A-antianalitik funksiya ya’ni $\frac{\partial A}{\partial z} = 0$ bunda $|A(z)| \le C < 1,\ \forall z \in D$,

Teorema 3. $R(x,y)$ – ratsional funksiya $L(a,r) \subset\subset D$ bo‘lsa, u holda quydagi tenglik o‘rinli bo‘ladi.

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$$\int_0^{2\pi} R(\cos\varphi,\sin\varphi)d\varphi = 2\pi \sum_{a_k\in L(a,r)} res_{\substack{A\\ z=a_k}} \frac{R\left(\frac{\psi^2(z,a)+r^2}{2r\psi(z,a)}, \frac{\psi^2(z,a)-r^2}{2ri\psi(z,a)}\right)}{\psi(z,a)}.$$

Isbot. a markazli r markazli leminskatalarni quydagi ko‘rinishda yozib olamiz. Quyidagi tenglik o‘rinli. Agar $|\psi(z,a)|=r$ bo‘lsa,

$$L(z,a)=\{|\psi(z,a)|\le r\} \qquad \psi(z,a)=re^{i\varphi} \quad \overline{\psi(z,a)}=re^{-i\varphi},$$

$$\overline{\psi(z,a)}=\frac{r^2}{\psi(z,a)},\ e^{i\varphi}=\frac{\psi(z,a)}{r},\ e^{-i\varphi}=\frac{\overline{\psi(z,a)}}{r}=\frac{r}{\psi(z,a)},$$

$$d\psi(z,a)=ire^{i\varphi}d\varphi, \quad d\psi(z,a)=i\psi(z,a)d\varphi,$$

$$d\varphi=\frac{d\psi(z,a)}{i\psi(z,a)},\ d\psi=dz+A(z)d\bar{z}$$

$$\cos\varphi=\frac{e^{i\varphi}+e^{-i\varphi}}{2}=\frac{\frac{\psi(z,a)}{r}+\frac{r}{\psi(z,a)}}{2}=\frac{\psi^2(z,a)+r^2}{2r\psi(z,a)}$$

$$\sin\varphi=\frac{e^{i\varphi}-e^{-i\varphi}}{2i}=\frac{\frac{\psi(z,a)}{r}-\frac{r}{\psi(z,a)}}{2i}=\frac{\psi^2(z,a)-r^2}{2ri\psi(z,a)}$$

bu tengliklarni $\int_0^{2\pi} R(\cos\varphi,\sin\varphi)d\varphi$ ifodaga qo‘yamiz.

$$\int_0^{2\pi} R(\cos\varphi,\sin\varphi)d\varphi=\int_{\gamma_r}\frac{R(\frac{\psi^2(z,a)+r^2}{2r\psi(z,a)},\frac{\psi^2(z,a)-r^2}{2ri\psi(z,a)})}{i\psi(z,a)}(dz+Ad\bar{z})=$$

$$=\frac{2\pi i}{i}\sum_{a_k\in L(a,r)} res_{\substack{A\\ z=a_k}}\frac{R(\frac{\psi^2(z,a)+r^2}{2r\psi(z,a)},\frac{\psi^2(z,a)-r^2}{2ri\psi(z,a)})}{\psi(z,a)}=$$

$$=2\pi\sum_{a_k\in L(a,r)} res_{\substack{A\\ z=a_k}}\frac{R(\frac{\psi^2(z,a)+r^2}{2r\psi(z,a)},\frac{\psi^2(z,a)-r^2}{2ri\psi(z,a)})}{\psi(z,a)}$$

demak yuqoridagi tenglik o‘rinli bo‘ladi.Teorema isbotlandi.

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