ISSN: 2181-1385
Scientific Journal Impact Factor (SJIF) 2021: 5.723
DOI: 10.24411/2181-1385-2021-00480

KVAZICHIZIQLI ISSIQLIK O`TKAZUVCHANLIK TENGLAMASI UCHUN QO`YILGAN BOSHLANG`ICH CHEGARAVIY MASALANI YECHISH

Mirjalol Turon o'g'li Shodmonqulov
Jizzax davlat pedagogika insitituti Ta’limda axborot texnologiyalari kafedrasi o’qituvchisi

Yigitali Xolmirza o'g'li Xandamov
Jizzax davlat pedagogika insitituti Ta’limda axborot texnologiyalari kafedrasi o’qituvchisi

ANNOTATSIYA

Maqolada ikkinchi tartibli parabolik tipdagi tenglamalar uchun chegaraviy masala qo`yilishi, nostatsionar issiqlik o`tkazuvchanlik tenglamasi uchun ayirmali sxemalar yaqinlashishi va maksimum prinsipi o`rganildi. Tanlab olingan namunaviy masala maksimum prinsipi va teks yaqinlashishga tekshirildi.

Kalit so’zlar: ikkinchi tartibli parabolik tipdagi tenglamalariga taluqli issiqlik o’tkazish tenglamasi bilan ifodalangan nostatsionar issiqlik o`tkazuvchanlik tenglamasi, chegaraviy masalaning qo’yilishi, ayirmali sxemalar, maksimum prinsipi va approksimatsiya.

SOLVING THE PRIMARY BOUNDARY PROBLEM FOR QUASIQUE HEAT CONDUCTION EQUATION

Mirjalol Turon ogli Shodmonkulov
Teacher of the Department of Information Technology in Education
Jizzakh State Pedagogical Institute

Yigitali Kholmirza ogli Khandamov
Teacher of the Department of Information Technology in Education
Jizzakh State Pedagogical Institute

ABSTRACT

The article addresses symmetric heating of a homogeneous plate using convection. This method solves the typical problem of optimal control. The problem of finding optimal control and minimum time in symmetric heating of a homogeneous plate is considered.

Keywords: Simple plate, heat transfer equation, convection, Fure criterion, optimal control, aproximation, maximum principle.

ISSN: 2181-1385
Scientific Journal Impact Factor (SJIF) 2021: 5.723
DOI: 10.24411/2181-1385-2021-00480

KIRISH

Issiqlik fizikasining asosiy masalasi bo'lib ikkinchi tartibli parabolik tipdagi tenglamalariga taluqli issiqlik o'tkazish tenglamasi bilan ifodalanadigan nostatsionar issiqlik o`tkazuvchanlik tenglamalarini tadqiq etish muhim ahamiyatga ega. Ushbu ishida ikkinchi tartibli parabolik tipdagi tenglamalar uchun chegaraviy masala qo`yilishi, nostatsionar issiqlik o`tkazuvchanlik tenglamasi uchun ayirmali sxemalar yaqinlashishi o`rganildi. Nostatsionar kvazichiziqli issiqlik o'tkazuvchanlik tenglamasi uchun qo`yilgan boshlang'ich chegaraviy masalada ayirmali sxemalar uchun maksimum prinsipi va tekis yaqinlashishi tekshirildi[1].

METODOLOGIYA

Quyidagi kvazichiziqli issiqlik o'tkazuvchanlik tenglamasi uchun birinchi chegaraviy masalani qaraymiz

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(k(u)\frac{\partial u}{\partial x}\right),\ 0<x<1,\ 0<t\le T, \tag{1}$$

$$u(x,0)=u_0(x),\quad u(0,t)=u_1(x),\quad u(1,t)=u_2(t), \tag{2}$$

bunda $k(u) > 0$[2].

Chiziqlimas tenglamada $k(u)$ funktsiyaning o'zgarish chegaralari ma'lum bo'lmasa, ko'pincha oshkor sxemalardan ochiladi. Shu sababli endi kvazichiziqli issiqlik o`tkazuvchanlik tenglamasi uchun ikki turdagi oshkormas vaznli ayirmali sxemalarni qaraymiz. Ulardan biri y^{j+1}, i=1,2,…,N-1 ga nisbatan chiziqli bo'lgan quyidagi a) sxemani qaraymiz:

$$\frac{y_i^{j+1}-y_i^j}{\tau}=\frac{\sigma}{h}\left[a_{i+1}\frac{y_{i+1}^{j+1}-y_i^{j+1}}{h}-a_i\frac{y_i^{j+1}-y_{i-1}^{j+1}}{h}\right]+$$

$$+\frac{1-\sigma}{h}\left[a_{i+1}\frac{y_{i+1}^{j}-y_i^{j}}{h}-a_i\frac{y_i^{j}-y_{i-1}^{j}}{h}\right] \tag{3}$$

bu yerda $a_i=0{,}5\left[k\left(y_i^j\right)+k\left(y_{i-1}^j\right)\right]$. Bu sxema absolyut turg'un va τ bo'yicha birinchi tartibli va h bo'yicha ikkinchi tartibli approksimatsiyaga ega. y^{j+1}, i=1,2,…,N-1 yechim progonka usuli bilan topiladi[7].

Ko'pincha quyidagi chiziqlimas b) sxema qo'llaniladi:

$$\frac{y_i^{j+1}-y_i^j}{\tau}=\frac{\sigma}{h}\left[a\left(y_{i+1}^{j+1}\right)\frac{y_{i+1}^{j+1}-y_i^{j+1}}{h}-a\left(y_i^{j+1}\right)\frac{y_i^{j+1}-y_{i-1}^{j+1}}{h}\right]+$$

$$+\frac{1-\sigma}{h}\left[a_{i+1}\frac{y_{i+1}^{j}-y_i^{j}}{h}-a_i\frac{y_i^{j}-y_{i-1}^{j}}{h}\right],\quad a\left(y_i^{j+1}\right)=\frac{k\left(y_i^{j+1}\right)+k\left(y_{i-1}^{j+1}\right)}{2}. \tag{4}$$

ISSN: 2181-1385
Scientific Journal Impact Factor (SJIF) 2021: 5.723
DOI: 10.24411/2181-1385-2021-00480

Bu sxemani amalga oshirish uchun u yoki bu iteratsion sxemani qo'llash zarur. Masalan bunday:

$$\frac{y_i^{(s+1)} - y}{\tau} = \frac{\sigma}{h}\left[a\left(y_{i+1}^{(s)}\right)\frac{y_{i+1}^{(s+1)} - y_i^{(s+1)}}{h} - a_i\left(y_i^{(s)}\right)\frac{y_i^{(s+1)} - y_{i-1}^{(s+1)}}{h}\right] +$$

$$+\frac{1-\sigma}{h}\left[a_{i+1}\frac{y_{i+1}^j - y_i^j}{h} - a_i\frac{y_i^j - y_{i-1}^j}{h}\right], \qquad (5)$$

$$s = 0,1,\ldots,M-1,\ \ y_i^{(0)} = y_i^j,\ \ y_i^{(M)} = y_i^{j+1}.$$

Bu yerda s – iteratsiya soni. Ko'rib turibmizki, chiziqlimas koeffitsiyentlar avvalgi iteratsiyadan olinyapdi, y_i^{j+1} uchun dastlabki yaqinlashish sifatida y_i^j tanlanyapdi. Yangi iteratsiyadagi $y_i^{(s+1)}$ qiymat (5) sistemadan progonka usuli yordamida topiladi. $M=1$ da (5) iteratsion sxema (3) ayirmali sxema bilan ustma-ust tushadi[4].

(2) chegaraviy shartlar quyidagicha approksimatsiya qilinadi:

$$y_i^0 = u_0(x_i),\quad y_0^{j+1} = \mu_1(t_j),\quad y_N^{j+1} = \mu_2(t_j). \qquad (6)$$

$y_i^{(s+1)}$ ga nisbatan ayirmali sxema chiziqlidir. (3), (4) iteratsion sxema bo`yicha hisoblashda yoki iteratsiya soni yoki iteratsiya yaqinlashishi aniqligi ε beriladi va quyidagi shart bajarilishi talab etiladi[4].

$$\max_i \left| y_{i+1}^{(s+1)} - y_i^{(s)} \right| \le \varepsilon.$$

MUHOKAMA VA NATIJALAR

Endi $k(u) = \chi_0 u^\delta$, $\chi_0 > 0, \delta > 0$ da (3), (4) vazinli sxemalarni σ ning turli qiymatlarida hisoblab sonli natijalar olamiz, quyidagi yechimni qaraymiz

$$u(x,t) = \begin{cases} \left[\delta D \chi_0^{-1}(Dt - x)\right]^{\frac{1}{\delta}}, & x \le Dt \ \ da, \\ 0, & x \ge Dt \ \ da, \end{cases}$$

bu yerda D – temperatura to'lqini tezligi. Bu funksiya quyidagi masalaning yechimi hisoblanadi

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\chi_0 u^\delta \frac{\partial u}{\partial x}\right),\ x > 0,\ t > 0, \qquad (7)$$

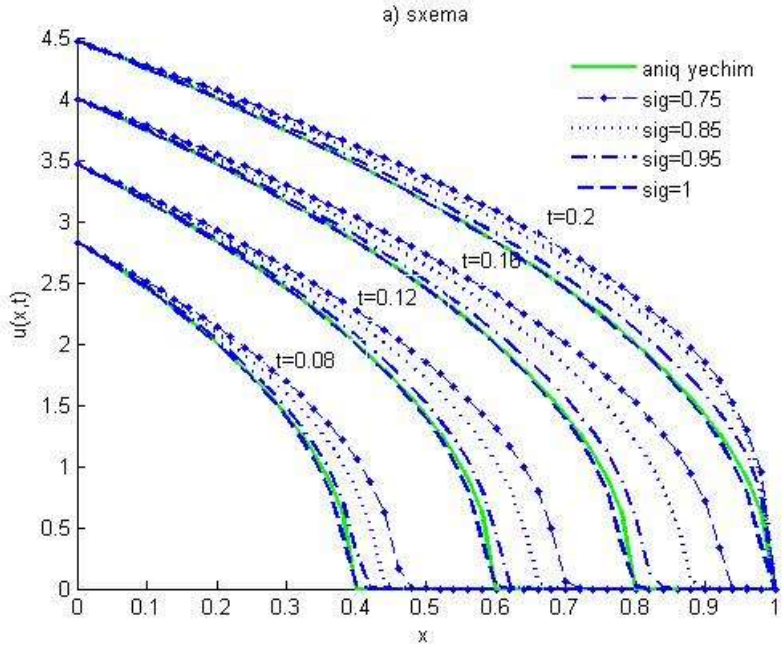
$$u(x,0) = 0,\ u(0,t) = u_0 t^{\frac{1}{\delta}},$$

ISSN: 2181-1385
Scientific Journal Impact Factor (SJIF) 2021: 5.723
DOI: 10.24411/2181-1385-2021-00480

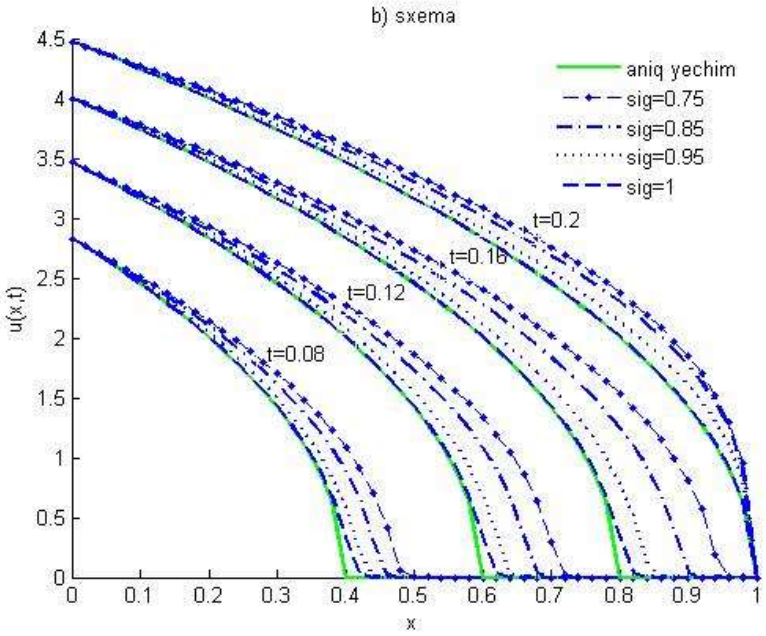
bu yerda $u_0 = (\frac{\delta D^2}{\chi_0})^{\frac{1}{\delta}}$.

XULOSA

Bu misolni σ parametrning qiymatlarida $\delta = 2, \chi_0 = 0{,}5, D = 5, h = 0{,}2$ (nuqtalar soni N=50) va $\tau = 2 \cdot 10^{-4}$ qadam bilan hisoblaymiz. Aniq yechim va (3), (4) sxemalar natijalari quyidagi 1, 2-rasmlarda keltirilgan[3].



1-rasm. (3) sxema bo`yicha olingan yechimlar



2-rasm. (4) sxema bo`yicha olingan yechimlar

ISSN: 2181-1385
Scientific Journal Impact Factor (SJIF) 2021: 5.723
DOI: 10.24411/2181-1385-2021-00480

REFERENCES

1. Самарский А.А., Вабищевич П.Н. Вычислительная теплопередача. –М.: Едиториал УРСС, 2003. – 784 с.
2. Хўжаров Б.Х. Қурилиш масалаларини сонли ечиш усуллари. – Т.: Ўзбекистон, 1995. – 272 б.
3. Дьяконов В. П. MATLAB 7.*/R2006/R2007: Самоучитель. – М.: ДМК Пресс, 2008. – 768 с.
4. Самарский А.А. Теория разностных схем. – М.: Наука, 1989. – 616 с.
5. Xandamov, Y. (2020). Система моделирования разрешения и совершенствования непрерывного образования. Архив Научных Публикаций JSPI.
6. Xandamov, Y., & Shodmonqulov, M. (2020). BIR JINSLI PLASTINKANI SIMMETRIK QIZDIRISH VAQTINI OPTIMALLASHTIRISH MASALASI. Архив Научных Публикаций JSPI.
7. Шодмонкулов, М. Т., & Хандамов, Й. Х. (2020). ВОПРОС ОПТИМИЗАЦИИ ВРЕМЕНИ СИММЕТРИЧНОГО НАГРЕВАНИЯ ОДНОРОДНОЙ ПЛАСТИНКИ. International scientific review, (LXX).